 Name: Solutions

Parabolas!

Remember that the definition of a parabola is the set of all points (x, y) equidistant from a fixed point (called the *focus*) and a fixed line (called the *directrix*).

In answering questions 1 and 2, you may use the result derived in class for a parabola with focus of $(0, p)$ and directrix of $y = -p$.

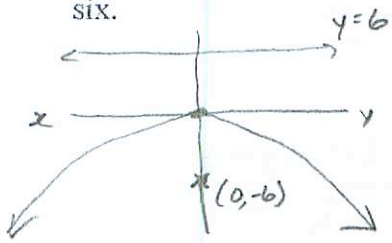
1. The *focal length* of a parabola is the distance from its focus to its vertex.

a. Find the focal length of the parabola $y = x^2$.

$y = \frac{1}{4p}x^2$ is the general parabola equation for a parabola with focus $(0,p)$ and directrix $y=-p$. So, for $y=x^2$, $\frac{1}{4p} = 1$ or $p = \frac{1}{4}$.

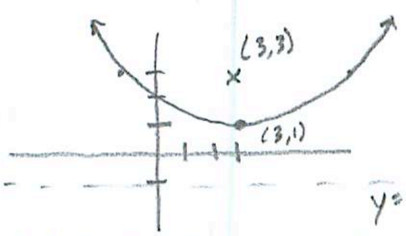
$\therefore$ Focal length is $1/4$.

b. Find the equation of the parabola with vertex at the origin, opening upside down, and a focal length of six.



$$y = -\frac{1}{24}x^2$$

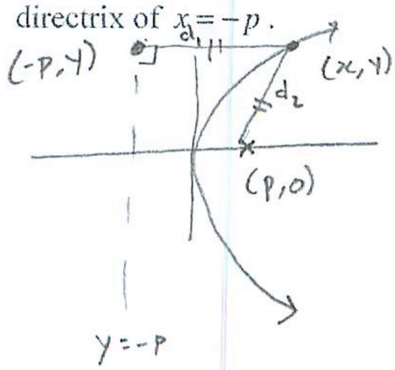
2. Find the focus and directrix of the parabola $y = \frac{1}{8}(x-3)^2 + 1$.



$p = 2$ 3 right 1 up

Focus: $(3,3)$ Directrix: $y=-1$

3. Let's find the equation of a parabola on its side! Find the equation of the parabola with focus of $(p, 0)$ and directrix of $x = -p$.



$$d_1 = d_2$$

$$\sqrt{(x-p)^2 + (y-0)^2} = x + p$$

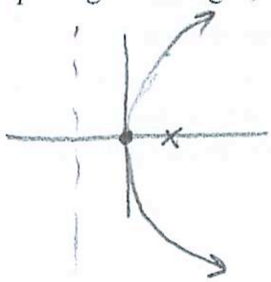
$$(x-p)^2 + y^2 = (x+p)^2$$

$$\cancel{x^2} - 2px + \cancel{p^2} + y^2 = \cancel{x^2} + 2px + \cancel{p^2}$$

$$y^2 = 4px$$

$$x = \frac{1}{4p}y^2$$

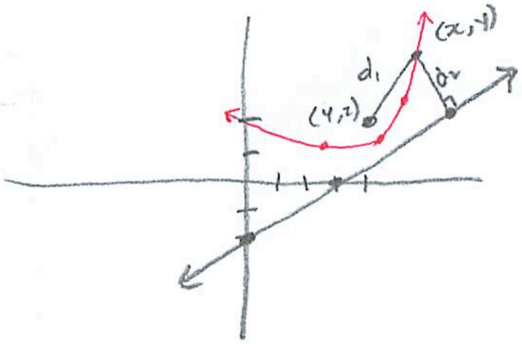
4. Use your equation from the previous question to find the equation of the parabola with vertex at the origin, opening to the right, and a focal length of 1.



$$x = \frac{1}{4}y^2$$

5. Let's find the equation of a parabola on an angle! Find the equation of the parabola with a focus of $(4,2)$ and directrix of $2x-3y-6=0$.

x	y
0	-2
3	0



$$d_1 = d_2$$

$$\sqrt{(x-4)^2+(y-2)^2} = \frac{|2x-3y-6|}{\sqrt{4+9}}$$

$$(x-4)^2+(y-2)^2 = \frac{(2x-3y-6)^2}{13}$$

$\boxed{\text{Extra Credit !!}}$ Find the equation of the parabola $y=x^2$ rotated $60°$ in a clockwise direction.